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Bohmian approach to spin-dependent time of arrival for particles in a uniform field and for particles passing through a barrier

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Abstract
The de Broglie–Bohm interpretation of quantum mechanics with and without spin-dependence is used to determine electron trajectories, arrival-time distribution of electrons and the mean arrival time of spin-$1/2$ particles (including electrons) in the presence of a uniform field and a barrier separately. The difference for the mean arrival times, which are calculated with different guidance equations, is examined versus mass of the arriving particle and versus group velocity of the arriving electron wave packet and also versus the width of the barrier. Numerical calculations show that these differences are of the order of 10^{-18}–10^{-17} s. Another feature of the modified guidance law is that Bohmian trajectories cross each other, but this does not contradict the single-valuedness of the wavefunction.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

In classical mechanics, each particle follows a definite trajectory, and so it is clear what one means by the time at which a particle arrives at a given place. In quantum mechanics, as opposed to classical physics, the meaning of the arrival time of a particle at a given location is not evident when the finite extent of the wavefunction and its spreading becomes relevant. Moreover, in the quantum case one expects an arrival-time distribution, and there are different proposals for this (see, e.g., the review article in [1], and the book [2]). As pointed out by Hannstein *et al* [3], these arrival-time distributions have been controversial, since they are derived from purely theoretical arguments without specifying a measurement procedure. The lack of a self-adjoint arrival-time operator conjugate to the free Hamiltonian lies at the core of the difficulties in the formulation of quantum arrival times [4]. In Bohm's interpretation of nonrelativistic quantum mechanics [5–11], an electron is a particle that has a well-defined trajectory and so the definition of arrival-time distributions is unambiguous. In Bohmain mechanics wave/particle duality means that a single electron is at all times both a point-like particle and a guiding wave. In Bohm's trajectory approach to the calculation of various characteristic times, it is only the particle component of the entity that is being clocked. If the arrival-time detector is not included in the Hamiltonian, then one has an expression for the unmeasured arrival-time distribution [12]. The arrival-time problem is unambiguously solved in the Bohmian mechanics, where for an arbitrary scattering potential $V(\mathbf{x})$, one finds [12–15] that for those particles that actually reach $\mathbf{x} = \mathbf{X}$, the arrival-time distribution is given by the modulus of the probability current density, i.e., $|\mathbf{J}(\mathbf{X}, T)|$. Even if a trajectory reaches X more than once, only one of its arrival times is equal to the specified value of T. Within Bohmian mechanics there is no paradox concerning the backflow effect because, according to guidance law $v = J/\rho$, particles arrive at X (only) from the right during any time interval when $J(X, T)$ is negative [1].

In nonrelativistic quantum mechanics, the form of the current density $\mathbf{J}$ is not uniquely determined by the continuity equation, a point which has been mentioned by a number of authors [16–18]. It is determined only up to a divergenceless vector. For instance, one can construct a new current $\mathbf{J}'$ by adding the divergenceless current $\mathbf{J}_s$ to the current $\mathbf{J}$. The newly defined current $\mathbf{J}' = \mathbf{J} + \mathbf{J}_s$ also satisfies the continuity equation, with the same probability density. In the case of spin-1/2 particles, Holland [19] showed that the particle current in the relativistic spin-1/2 Dirac theory is unique. Demanding that the non-relativistic spin-1/2 particle current be obtained from the nonrelativistic limit of the Dirac current, this nonrelativistic current is also unique. Struyve *et al* [20] have considered the uniqueness of paths for spin-0 and spin-1 particles. The spin-dependent Bohm trajectories have been investigated for hydrogen eigenstates [21] and an electronic transition in hydrogen [22].

It has been argued that for free spin eigenstates, spin contributions would in principle be experimentally distinguishable for arrival-time distributions of spinless and spin-1/2 particles [23]. Holland *et al* [24] have explored some of the implications of the modified guidance law in the case of the two-slit quantum interference. In this case, the trajectories cross each other and they also cross the symmetry axis.

The aim of the present paper is to explore some of the implications of the revised nonrelativistic guidance equation. Section 2 contains a very brief review of relevant parts of Bohm's interpretation of quantum mechanics. In section 3, by using the nonrelativistic limit of the Dirac current density, numerical computations of the effect of the spin-dependent term on the arrival time at a given location are presented for both a Gaussian wave in a uniform field and a Gaussian wave passing through a barrier.

2. Bohm's trajectory interpretation of quantum mechanics and the arrival-time distributions

In nonrelativistic Bohmian mechanics the world is described by point-like particles which follow trajectories determined by a law of motion. The evolution of the positions of these particles are guided by a wavefunction which itself evolves according to the Schrödinger equation. Bohmian mechanics makes the same predictions as the ordinary nonrelativistic quantum mechanics for the results of any experiment, provided we assume a random distribution for configuration of the system and the apparatus at the beginning of the experiment, given by $\rho(\mathbf{x}, 0) = \Psi^{\dagger}(\mathbf{x}, 0)\Psi(\mathbf{x}, 0)$. If the probability density for the configuration satisfies $\rho(\mathbf{x}, t_0) = \Psi^\dagger(\mathbf{x}, t_0)\Psi(\mathbf{x}, t_0)$ at some initial time t_0, then the density to which this is carried by the continuity equation at any time t is also given by $\rho(\mathbf{x}, t) = \Psi^\dagger(\mathbf{x}, t)\Psi(\mathbf{x}, t)$ [25]. As most of the quantum measurements boil down to position measurements, Bohm's theory and the standard quantum mechanics generally yield the same detection probabilities. The situation is different, however, if one considers, for example, measurements involving time-related quantities, such as arrival times, tunnelling times, etc., Bohm's theory makes unambiguous predictions for such measurements, but there is no consensus about what these quantities should be in the conventional quantum mechanics [26]. Given the initial position $\mathbf{x}^{(0)} \equiv \mathbf{x}(t = 0)$ of a particle with the initial wavefunction $\Psi(\mathbf{x}, t = 0)$, its subsequent trajectory $\mathbf{x}(\mathbf{x}^{(0)}, t)$ is uniquely determined by the simultaneous integration of the time-dependent Schrödinger equation, and the guidance equation $\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = \mathbf{v}(\mathbf{x}(t), t)$, in which $\mathbf{v} = \frac{\mathbf{J}}{\rho}$. For spinless particles, the Schrödinger equation yields

$$\mathbf{J}(\mathbf{x}, t) = (\hbar/m)\mathrm{Im}(\psi^*(\mathbf{x}, t)\nabla\psi(\mathbf{x}, t)). \tag{1}$$

But, for systems with more than one spatial dimension the probability current density, and hence the particle equation of motion, is not uniquely defined within nonrelativistic quantum mechanics. But, Holland [19] has shown that the probability current density deduced from the continuity equation is uniquely defined for Dirac electrons, when Lorentz covariance is imposed. Taking the nonrelativistic limit for a spin eigenstate in the absence of a magnetic field, one gets

$$\begin{aligned}\mathbf{J}(\mathbf{x}, t; \widehat{\mathbf{s}}) &= \mathbf{J}(\mathbf{x}, t) + \frac{1}{m}\nabla\rho(\mathbf{x}, t) \times \mathbf{s} \\ &= (\hbar/m)[\mathrm{Im}(\psi^*(\mathbf{x}, t)\nabla\psi(\mathbf{x}, t)) + \mathrm{Re}(\psi^*(\mathbf{x}, t)\nabla\psi(\mathbf{x}, t)) \times \widehat{\mathbf{s}}],\end{aligned} \tag{2}$$

where

$$\mathbf{s} = \frac{\hbar}{2}\widehat{\mathbf{s}} = \frac{\hbar}{2}\chi^\dagger\widehat{\sigma}\chi, \tag{3}$$

is the spin vector associated with the spin eigenstate χ, which is a two-component spinor normalized to unity ($\chi^\dagger\chi = 1$) and $\rho = \psi^*\psi$. The nonrelativistic particle is, in effect, guided by the two-component wavefunction $\Psi(\mathbf{x}, t; \mathbf{s}) \equiv \psi(\mathbf{x}, t)\chi$, and the original expression for the nonrelativistic velocity field should then be replaced by

$$\mathbf{v}(\mathbf{x}, t; \widehat{\mathbf{s}}) \equiv \frac{\mathbf{J}(\mathbf{x}, t; \widehat{\mathbf{s}})}{|\psi(\mathbf{x}, t)|^2} = \mathbf{v}(\mathbf{x}, t) + \frac{1}{m}\nabla\log\rho \times \mathbf{s}, \tag{4}$$

in which $\mathbf{v}(\mathbf{x}, t) = \frac{\mathbf{J}(\mathbf{x},t)}{\rho}$. For a 3D system with a wavefunction which is in the factorized form $\psi(\mathbf{x}, t) = \psi_x(x, t)\psi_y(y, t)\psi_z(z, t)$ and for $\widehat{\mathbf{s}} = (0, 0, 1)$, equation (4) takes the form

$$\begin{aligned} v_x &= \frac{\hbar}{m}\left[\mathrm{Im}\left(\frac{\partial_x\psi_x}{\psi_x}\right) + \mathrm{Re}\left(\frac{\partial_y\psi_y}{\psi_y}\right)\right], \qquad v_y = \frac{\hbar}{m}\left[\mathrm{Im}\left(\frac{\partial_y\psi_y}{\psi_y}\right) - \mathrm{Re}\left(\frac{\partial_x\psi_x}{\psi_x}\right)\right], \\ v_z &= \frac{\hbar}{m}\left[\mathrm{Im}\left(\frac{\partial_z\psi_z}{\psi_z}\right)\right], \end{aligned} \tag{5}$$

where we have used the notation $\partial_x\psi_x = \frac{\mathrm{d}\psi_x}{\mathrm{d}x}$ and so on. A striking property of the spin-dependent term is that the components of the particle motion in orthogonal directions are generally mutually dependent, even when the wavefunction factorizes in these directions. For a nonrelativistic guiding wave of the form $\Psi(\mathbf{x}, t; \widehat{\mathbf{s}}) = \psi(\mathbf{x}, t)\chi$, with $\psi(\mathbf{x}, t)$ being a solution of the Schrödinger equation and χ being a fixed spinor, Bohm's trajectory result for the distribution of particle arrival times T, at a given location $\mathbf{x} = \mathbf{X}$, is given by [12]

$$\Pi(T, \mathbf{X}; \widehat{\mathbf{s}}) = |\mathbf{J}(\mathbf{X}, T; \widehat{\mathbf{s}})| \Big/ \int_0^\infty \mathrm{d}t|\mathbf{J}(\mathbf{X}, t; \widehat{\mathbf{s}})|. \tag{6}$$

The mean arrival time of the particles reaching a detector located at $\mathbf{X}$ is given by

$$\tau = \int_0^\infty \mathrm{d}t\, t\,\Pi(t, \mathbf{X}; \widehat{\mathbf{s}}) = \frac{\int_0^\infty \mathrm{d}t\, t|\mathbf{J}(\mathbf{X}, t; \widehat{\mathbf{s}})|}{\int_0^\infty \mathrm{d}t|\mathbf{J}(\mathbf{X}, t; \widehat{\mathbf{s}})|}, \tag{7}$$

in the presence of the spin-dependent contribution and

$$\tau_i = \int_0^\infty \mathrm{d}t\, t\,\Pi_i(t, \mathbf{X}) = \frac{\int_0^\infty \mathrm{d}t\, t|\mathbf{J}(\mathbf{X}, t)|}{\int_0^\infty \mathrm{d}t|\mathbf{J}(\mathbf{X}, t)|}, \tag{8}$$

in the absence of the spin-dependent contribution. As pointed out by several earlier authors [29–31] the integral in the numerator of equation (8) diverges formally. In our examples, below, numerical calculations show that $|\mathbf{J}|$ decays faster than t^{-2} and thus the integral converges. It must be mentioned that the definition of the mean arrival time used in equation (8) is not a uniquely derivable result within the standard quantum mechanics. We compute the arrival-time distribution and the mean arrival time at a given location for two problems: (1) a symmetrical Gaussian packet in a uniform field and (2) a symmetrical Gaussian packet passing through a 1D barrier with and without the spin contribution. Using the Runge–Kutta method for the integration of the guidance law, Bohmian paths of these problems have also been computed.

2.1. Symmetric Gaussian packet in a uniform field

For the potential $V(\mathbf{x}) = \mathbf{K}\cdot\mathbf{x}$, the wavefunction and the probability density at time t are given by [8]

$$\begin{aligned}
\psi(\mathbf{x}, t) &= \left(2\pi s_t^2\right)^{-3/4} \exp\left\{-(\mathbf{x} - \mathbf{u}t + \mathbf{K}t^2/2m)^2/4s_t\sigma_0\right.\\
&\qquad \left.+ (\mathrm{i}m/\hbar)\left[(\mathbf{u} - \mathbf{K}t/m)\cdot\left(\mathbf{x} - \frac{1}{2}\mathbf{u}t\right) - \mathbf{K}^2t^3/6m^2\right]\right\},\\
\rho(\mathbf{x}, t) &= (2\pi\sigma^2)^{-3/2} \exp\left\{-\frac{\left[\mathbf{x} - \mathbf{u}t + \frac{\mathbf{K}t^2}{2m}\right]^2}{2\sigma^2}\right\},
\end{aligned} \tag{9}$$

provided that this symmetrical Gaussian wavefunction is centred around the origin at $t = 0$. In equation (9), $s_t = \sigma_0\left(1 + \mathrm{i}\hbar t/2m\sigma_0^2\right)$, $\sigma = \sigma_0\left[1 + \left(\frac{\hbar t}{2m\sigma_0^2}\right)^2\right]^{1/2}$ and $\mathbf{u}$ is the group velocity. Equation (9) is in the factorized form in three coordinate directions. We choose the uniform field and the group velocity to be in the x-direction, i.e., $V(\mathbf{x}) = \mathbf{K}\cdot\mathbf{x} = Kx$ and $\mathbf{u} = (u, 0, 0)$ and the spin vector in the z-direction. The speed of the wave packet's centre in the y and z directions has been set equal to zero and therefore this function will spread but will not propagate in these directions. Using equation (5), one gets for this factorized wave

$$\begin{aligned}
v_x &= u - \frac{Kt}{m} + \frac{\hbar^2 t}{4m^2\sigma_0^4 + \hbar^2t^2}\left(x - ut + \frac{Kt^2}{2m}\right) - \frac{\hbar}{2\sigma^2}y,\\
v_y &= \frac{\hbar^2 t}{4m^2\sigma_0^4 + \hbar^2t^2}y + \frac{\hbar}{2\sigma^2}\left(x - ut + \frac{Kt^2}{2m}\right),\\
v_z &= \frac{\hbar^2 t}{4m^2\sigma_0^4 + \hbar^2t^2}z,
\end{aligned} \tag{10}$$

from which one can compute Bohmian trajectories. In the z-direction, one gets $z(t) = z_0\left(1 + \frac{\hbar^2t^2}{4m^2\sigma_0^4}\right)^{1/2}$, in which z_0 is the z-component of the initial position of the particle. Using the guidance law with equation (10) one can easily get the probability current density,

$$\mathbf{J}(\mathbf{x},t) = (2\pi\sigma^2)^{-3/2}\exp\left\{-\frac{\left(x-ut+\frac{Kt^2}{2m}\right)^2+y^2+z^2}{2\sigma^2}\right\}$$
$$\times\left[\left(u-\frac{Kt}{m}+\frac{\hbar^2 t}{4m^2\sigma_0^4+\hbar^2t^2}\left(x-ut+\frac{Kt^2}{2m}\right)\right)\widehat{\mathbf{x}}\right.$$
$$\left.+\left(\frac{\hbar^2 t}{4m^2\sigma_0^4+\hbar^2t^2}y\right)\widehat{\mathbf{y}}+\left(\frac{\hbar^2 t}{4m^2\sigma_0^4+\hbar^2t^2}z\right)\widehat{\mathbf{z}}\right], \tag{11}$$

$$\mathbf{J}(\mathbf{x},t;\widehat{\mathbf{s}}) = (2\pi\sigma^2)^{-3/2}\exp\left\{-\frac{\left(x-ut+\frac{Kt^2}{2m}\right)^2+y^2+z^2}{2\sigma^2}\right\}$$
$$\times\left[\left(u-\frac{Kt}{m}+\frac{\hbar^2 t}{4m^2\sigma_0^4+\hbar^2t^2}\left(x-ut+\frac{Kt^2}{2m}\right)-\frac{\hbar}{2\sigma^2}y\right)\widehat{\mathbf{x}}\right.$$
$$+\left(\frac{\hbar^2 t}{4m^2\sigma_0^4+\hbar^2t^2}y+\frac{\hbar}{2\sigma^2}\left(x-ut+\frac{Kt^2}{2m}\right)\right)\widehat{\mathbf{y}}$$
$$\left.+\left(\frac{\hbar^2 t}{4m^2\sigma_0^4+\hbar^2t^2}z\right)\widehat{\mathbf{z}}\right]. \tag{12}$$

2.2. Symmetric Gaussian packet passing through a barrier in the x-direction

Consider a 3D system with a planar and time-independent potential $V(\mathbf{x}) = V_0$ for $0 \leqslant x \leqslant d$ and zero otherwise and a wavefunction which has the factorable form $\psi(\mathbf{x},t) = \psi_x(x,t)\psi_y(y,t)\psi_z(z,t)$[3] with $\psi_x(x,t)$ satisfying $\mathrm{i}\hbar\partial\psi_x/\partial t = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V_0\right]\psi_x(x,t)$ and $\psi_y(y,t)$ and $\psi_z(z,t)$ satisfying the corresponding free-particle Schrödinger equations. It is assumed that $\psi_y(y,0)$ and $\psi_z(z,0)$ are Gaussian packets,

$$\psi_y(y,t) = \frac{1}{\left(2\pi s_t^2\right)^{1/4}}\,\mathrm{e}^{-y^2/4s_t\sigma_0}, \qquad \psi_z(z,t) = \frac{1}{\left(2\pi s_t^2\right)^{1/4}}\,\mathrm{e}^{-z^2/4s_t\sigma_0}. \tag{13}$$

Again, the speed of the wave packet's centre in the y and z directions has been set equal to zero and therefore this function will spread but not propagate in these directions. Once a particle incident from the left and prepared in the state $\left(2\pi\sigma_0^2\right)^{-1/4}\exp\left\{-\left(\frac{x-x_0}{2\sigma_0}\right)^2+\mathrm{i}k_0(x-x_0)\right\}$ at $t=0$ has passed completely through the barrier, the transmission probability amplitude (transmitted part of the wavefunction) is [1]

$$\psi_x(x,t) = \frac{1}{\sqrt{2\pi}}\int_0^\infty \mathrm{d}k\,\mathrm{e}^{\,\mathrm{i}kx-\mathrm{i}k^2\hbar t/(2m)}T(k)\tilde{\psi}(k), \tag{14}$$

in which

$$\tilde{\psi}(k) = \left(\frac{2\sigma_0^2}{\pi}\right)^{1/4}\mathrm{e}^{-\sigma_0^2(k-k_0)^2}\mathrm{e}^{-\mathrm{i}kx_0}, \tag{15}$$

where x_0 is the centroid of $|\psi(x,t=0)|^2$ and k_0 is the centroid of $|\tilde{\psi}(k)|^2$, which is related to the group velocity by $\mathbf{u} = \hbar/m(k_0,0,0)$, and

$$T(k) = \frac{2kq}{2kq\cos(qd)-\mathrm{i}(q^2+k^2)\sin(qd)}\,\mathrm{e}^{-\mathrm{i}kd}; \qquad \hbar^2k^2\big/2m \neq V_0,$$

[3] The factorizable wavefunction $\psi(\mathbf{x},t) = \psi_x(x,t)\psi_y(y,t)\psi_z(z,t)$ is a possible solution to the Schrödinger wave equation when the potential is of the form $V(\mathbf{x}) = V_x(x)+V_y(y)+V_z(z)$ (including when $V_y(y) = 0 = V_z(z)$) where, in general, the factorizability will be preserved for all t (if it is true at one instant) only if V is time-independent.

$$T(k) = \frac{2}{2 + \mathrm{i}kd}\,\mathrm{e}^{-\mathrm{i}kd}; \qquad \hbar^2k^2\big/2m = V_0, \tag{16}$$

is the transmission probability amplitude for monochromatic incidence with $q = \sqrt{2m(\hbar^2k^2/2m - V_0)}/\hbar$. In this case because we do not have an analytical expression for $\psi_x(x,t)$, we will not have an analytical expression for the probability density and for the current probability density. Using the rectangular approximation for the integration, the wavefunction at the detector location is computed for each instant of time from equations (13) and (14). It should be noted that, because of the factor $\tilde{\psi}(k)$ in the integrand of equation (14), the range of the integral is practically confined to $[0, k_0 + 3\sigma_k]$. Then, using equation (5) as well as the guidance law, the probability current density is computed and finally by using equation (6), the arrival-time distribution is calculated. For the motion in the z-direction, one gets

$$z(t) = z_0\left(1 + \frac{\hbar^2t^2}{4m^2\sigma_0^4}\right)^{1/2}, \tag{17}$$

in which z_0 is the z-component of the initial position of the particle.

3. Numerical results

Numerical calculations are presented for a case of symmetric Gaussian packet in a uniform field and for a case of symmetric Gaussian packet passing through a barrier in the x-direction. We compute the mean arrival time and the arrival-time distribution with and without the spin-dependent term. Using the Runge–Kutta method for the integration of guidance equation, we compute some Bohmian paths for each case.

3.1. Symmetric Gaussian packet in a uniform field

The initial wave packet is peaked at $(x_0 = 0, y_0 = 0, z_0 = 0)$, with $\sigma_0 = 5$ Å. The detector position is chosen at $(x = 20, y = 20, z = 20)$ Å. Figure 1 shows the arrival-time distribution of electrons for $K = mg$, corresponding to particles in a gravity field, with $E_0 = \frac{1}{2}mu^2 = 5$ eV and $g = 9.8$ m s^{-2}.

Figure 2 shows the mean arrival time versus the mass of the arriving particle for a fixed value of E_0 ($E_0 = 5$ eV). From this figure, it follows that the spin-dependent term increases the mean arrival time ($\tau > \tau_i$) and has a very small effect on the mean arrival time at the given location, and the quantity $\tau - \tau_i$ decreases with mass.

Figure 3 shows the mean arrival time of electrons versus the ratio of the group velocity to the light velocity u/c. (Note that we have fixed the mass.) It follows from this figure that the spin-dependent term increases the arrival time ($\tau > \tau_i$) and has a very small effect on the mean arrival time at the given location, and that the quantity $\tau - \tau_i$ increases with the initial group velocity at first and then decreases with it.

Figure 4 shows some Bohmian paths for electrons. One can see that some trajectories cross each other when one considers the spin-dependent term. It should be noted that trajectories cross each other for different values of the y-coordinate. Consequently, there is no problem with the single-valuedness of the wavefunction. The spin-dependent term can also change the fate of the individual trajectories. There are some trajectories that do not reach the detector location, with the spin-dependent term, but reach the detector location without this term.

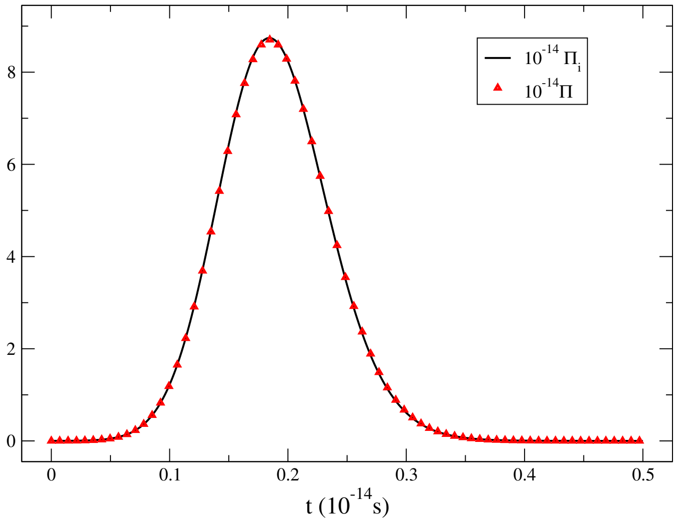

Figure 1. The arrival-time distribution of electrons versus time, without the spin-dependent term (solid curve) and with the spin-dependent term (shown by triangles), for a symmetric Gaussian packet in a uniform field.

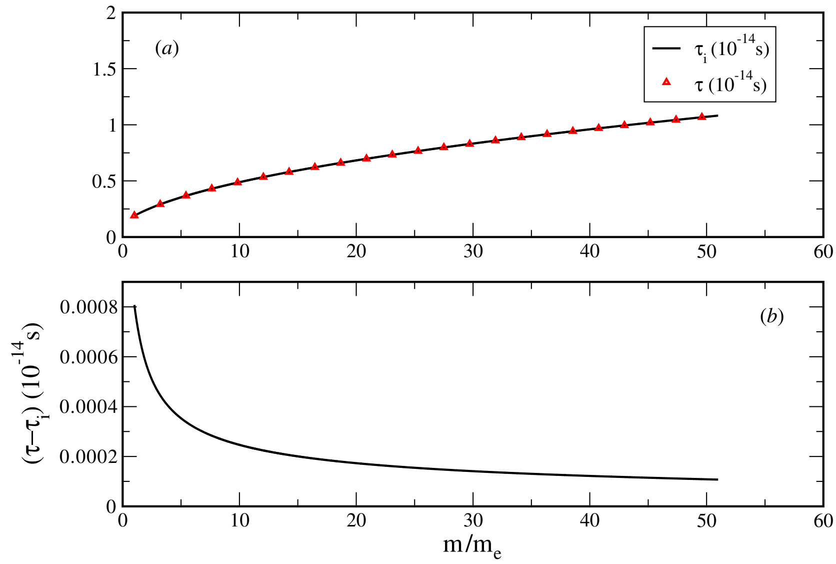

Figure 2. (*a*) The mean arrival time versus the ratio of the mass of the arriving particles to the mass of electron, without the spin-dependent term (solid curve) and with the spin-dependent term (shown by triangles) and (*b*) the difference between τ_i and τ versus m/m_e for a symmetric Gaussian packet in a uniform field.

3.2. Symmetric Gaussian packet passing through a barrier in the x-direction

The centroid of $|\psi_x(x,0)|^2$, x_0, must be chosen far enough to the left of the barrier so that $|\psi_x(x,0)|^2$ is negligible in the region $x > 0$. So we choose x_0 to be $-10\sigma_0$ with $\sigma_0 = 5$ Å. The detector location is chosen at $(x = 20, y = 20, z = 20)$ Å.

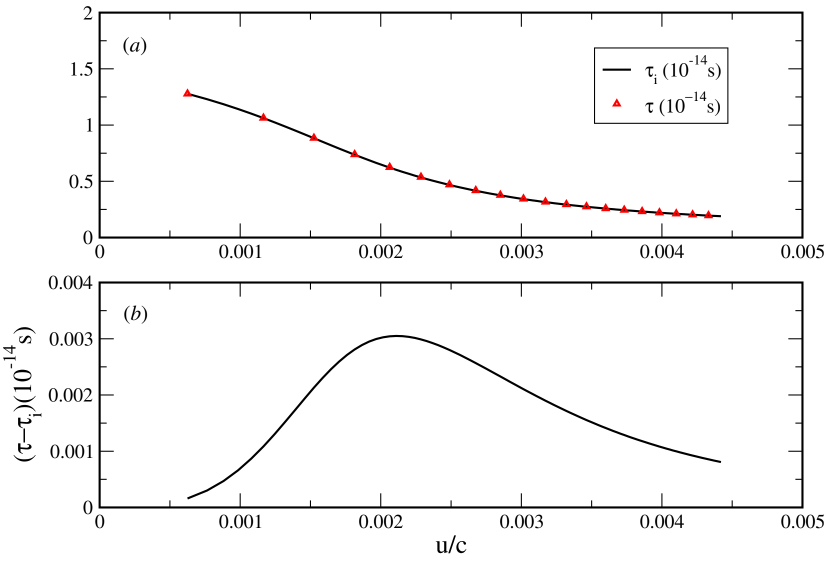

Figure 3. (*a*) The mean arrival time of electrons versus the ratio of the group velocity to the light velocity, without the spin-dependent term (solid curve) and with the spin-dependent term (shown by triangles) and (*b*) the difference between τ_i and τ versus u/c for a symmetric Gaussian packet in a uniform field.

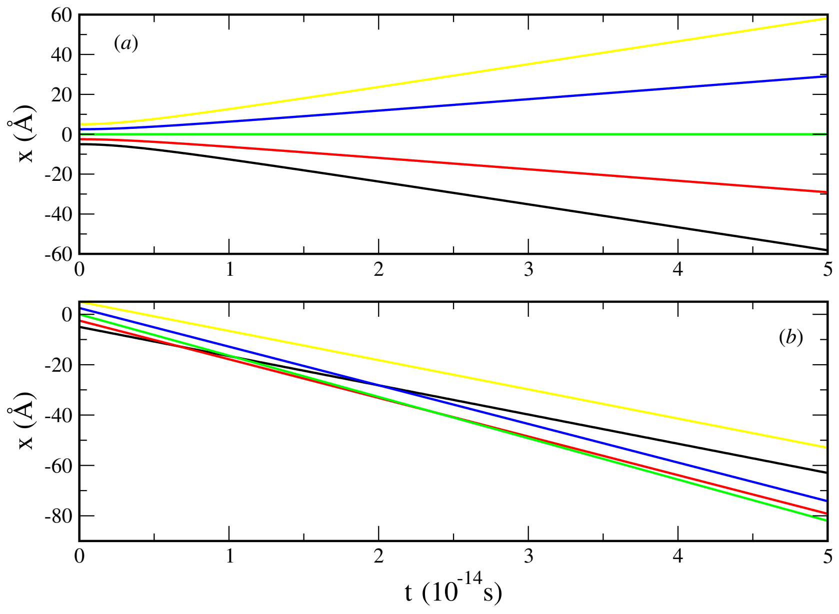

Figure 4. Some Bohmian trajectories of electrons in *x*–*t* plane with the initial position $(x^{(0)}, y^{(0)} = \sqrt{2\sigma_0^2 - x^{(0)2}}, z^{(0)} = 0)$: (*a*) in the absence of the spin-dependent term and (*b*) in the presence of the spin-dependent term, for a symmetric Gaussian packet in a uniform field.

Figure 5 shows the arrival-time distribution of electrons, for $V_0 = 8$ eV, $E_0 = \frac{1}{2}mu^2 = 10$ eV and $d = 10$ Å.

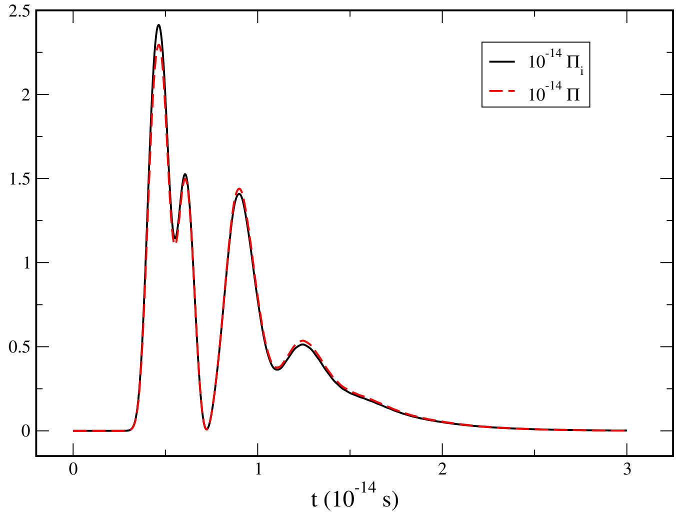

Figure 5. The arrival-time distribution of electrons versus time, without the spin-dependent term (solid curve) and with the spin-dependent term (dashed curve), for a symmetric Gaussian packet passing through a barrier in the x-direction.

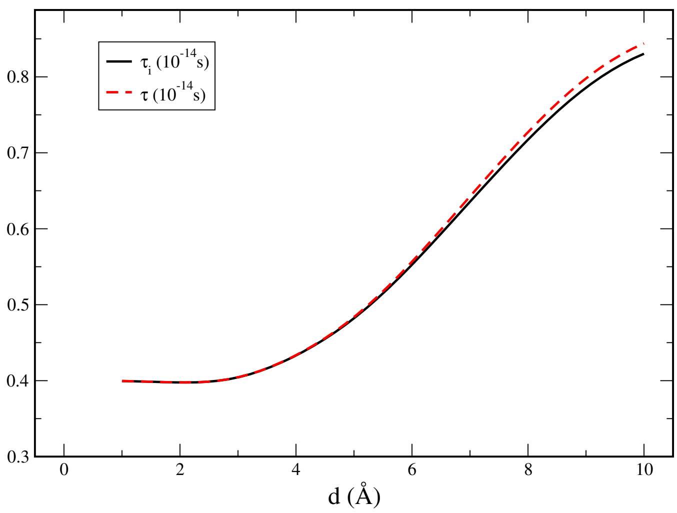

Figure 6. The mean arrival time of electrons versus the width of the barrier, without the spin-dependent term (solid curve) and with the spin-dependent term (dashed curve), for a symmetric Gaussian packet passing through a barrier in the x-direction.

Figure 6 shows the mean arrival time of electrons at the given location, versus width of the barrier. From this figure, it follows that the spin-dependent term increases the mean arrival time ($\tau > \tau_i$) and the quantity $\tau - \tau_i$ increases with the width of the barrier.

Figure 7 shows the mean arrival time of electrons versus the ratio of the group velocity to the light velocity u/c for $d = 10$ Å. It follows from this figure that the spin-dependent term increases the arrival time ($\tau > \tau_i$) and has a very small effect on the mean arrival time at the specified location, and that the quantity $\tau - \tau_i$ decreases with the initial group velocity. Note

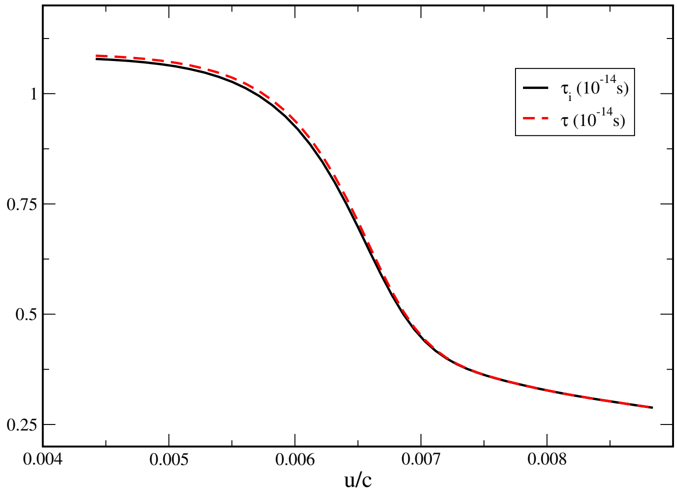

Figure 7. The mean arrival time of electrons versus the ratio of the group velocity to the light velocity, without the spin-dependent term (solid curve) and with the spin-dependent term (dashed curve), for a symmetric Gaussian packet passing through a barrier in the x-direction.

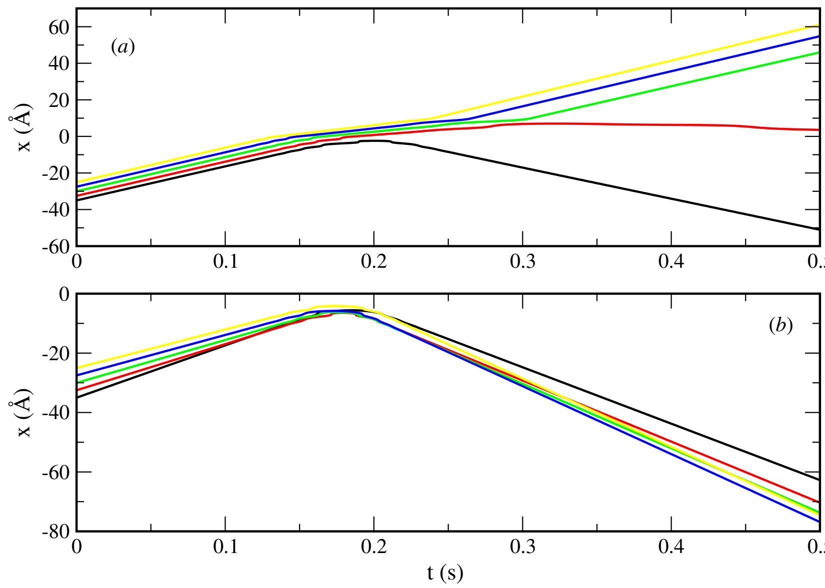

Figure 8. Some Bohmian trajectories of electrons in x–t plane with the initial position $(x^{(0)}, y^{(0)} = \sqrt{50\sigma_0^2 - x^{(0)2}}, z^{(0)} = 0)$: ($a$) in the absence of the spin-dependent term and (b) in the presence of the spin-dependent term, for a symmetric Gaussian packet passing through a barrier in the x-direction.

that we could not decrease the initial energy (or the initial group velocity) as we wanted, as in figure 3, because if $E_0 = \frac{1}{2}mu^2 \ll V_0$, then the transmission probability would be very small and no particle can cross the barrier to reach the detector position.

Figure 8 shows some Bohmian paths for electrons. One can see that trajectories cross each other (figure 8), when one considers the spin-dependent term. As figure 4 they cross each other for different values of the y-coordinate. The spin-dependent term can also change the fate of the individual trajectories. There are some trajectories that do not reach the barrier, with the spin-dependent term, but they cross the barrier without this term. However, ensemble averaging yields the same transmission and reflection probabilities.

4. Summary and discussion

To summarize, we have studied the problem of the arrival time for both a symmetrical Gaussian wave packet incident on a rectangular barrier and a Gaussian packet in a uniform gravitational field in 3D within Bohm's causal interpretation of quantum mechanics. We have employed both original guidance law $\mathbf{v} = \mathbf{J}/\rho$ and an additional spin-dependent term $\nabla \log \rho \times \mathbf{s}$ in the equation of motion, where $\rho = \psi^*\psi$. The spin-1/2 particle (including electron) is assumed to be in a spin-up eigenstate, with associated spin vector $\mathbf{s} = \frac{\hbar}{2}\widehat{\mathbf{k}}$ throughout this paper. We have examined dependence of the mean arrival time at a given location versus several parameters: m/m_e, u/c and d. In all cases spin-dependence increases the mean arrival time, i.e., $\tau > \tau_i$. The difference for the mean arrival times, which are calculated with different guidance equations, i.e., $\tau - \tau_i$, is of the order of 10^{-18}–10^{-17} s and has a monotonic behaviour versus mass of the arriving particle and the width of the barrier: it decreases with mass but increases with the width of the barrier, but it does not have monotonic behaviour versus the group velocity of the initial packet (at least in the range of our parameters). The effect of the spin term on quantum trajectories has also been considered. We have plotted some Bohmian paths in the x–t plane. The trajectories are seen to cross as a result of spin-dependence, but this is not a violation of the single-valuedness of the wavefunction since they reach the same point in x–t plane at different values of the y-coordinate. The spin-dependent term can also change the fate of the individual trajectories. There are some trajectories that do not reach the barrier with the spin-dependent term, but they cross the barrier without this term.

It has been argued that for free spin eigenstates, spin contributions would in principle be experimentally distinguishable for arrival-time distributions of spinless and spin-1/2 particles [23]. Within Bohm's causal theory of quantum mechanics the *unmeasured* arrival-time distribution is given by the modulus form of the current density. Therefore, the calculations presented here are for the arrival time in the absence of any measuring device. In this interpretation if we try to measure properties other than *position* (the only intrinsic property), we find that the result is affected by the process of interaction in a way that depends, not only on the total wavefunction but also on the details of the initial conditions of both the particle and the apparatus. As argued by McKinnon *et al* [27] concerning the delay time and by Leavens [28] concerning the traversal time, even if the arrival-time distribution can be determined *experimentally*, the theoretical causal expressions for it will be of relevance to the experiment only if the measuring process does not strongly perturb the unmeasured quantity. One has to devise a suitable experimental set-up to determine the mean arrival times for a particular context. Our calculations can be verified if mean arrival times can be experimentally measured in such a way that the experiment does not perturb the unmeasured quantity. It may be that the *causal* arrival times for an unperturbed system have little or no relation to measured times.

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